

# Combined technique of MMSE Estimator and Kalman Filter to suppress CFO Effect in LTE Uplink Transmission

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**Abstract** – The third generation partnership project (3GPP) Long Term Evolution (LTE) is the newest technology to deal with the increasing need of mobile broadband services. Single-Carrier Frequency-Division Multiple Access (SC-FDMA) technology is being used for uplink (UL) transmission. But SC-FDMA system is also susceptible to CFO, comparable with other Orthogonal Frequency Division Multiple (OFDM) based systems which is generally occurs due to Doppler shift and oscillator mismatch. The numerous CFOs of the received signals will disrupt the orthogonality between the various subcarriers. Consequently, the generation of ICI and MAI into the received signals will occur. Thus the performance of the system will be sincerely degraded. To suppress the effect of CFO, the new hybrid technique of MMSE Estimator and Kalman Filter is proposed. Simulations results show that the algorithm is effective in the presence of CFO. The value of BER diminishes to some extent by applying proposed technique and results in significant improvement in the system performance.

**Index Terms** – CFO, LTE Uplink, Kalman Filter, MMSE.

## 1. INTRODUCTION

In the last few decades the field of wireless communications has gained a huge development. Due to the constant increase of mobile subscribers, the need of an improved data rate is also emergent. Big efforts of research on various efficient transmission techniques are also being carried out to meet the requirements of high data transmission rates. Various improved multiple access methods are suggested to manage the increased number of subscribers. Today main emphasis is laid on multicarrier systems. In multicarrier systems the entire frequency band is divided into a set of non-overlapping sub channels [8]. OFDM has been applied to various wire line and wireless communication systems including ADSL, VDSL, wireless LAN due to its robustness to the frequency selective fading and is considered as a vital transmission technology for the next generation wireless communication systems [9]. In OFDM, a set of modulated subcarriers are multiplexed in the time domain thus making it a multi-carrier narrowband system. The drawback is that the signal has a high amplitude variability called Peak to Average Power Ratio (PAPR), which reduces

the efficiency of the transmitter power. This forces us to invent new technology for high data rate communication.

Recently an attractive single carrier transmission and multiple access scheme is evolved which provide a comprehensive IP-based integrated solution to users at an affordable price and at higher data rates than the previous generations [4]. This can be accomplished after the convergence of all types of wired and wireless technologies and will be capable of providing data rates between 100 Mbps and 1Gbps with premium quality and high security. Spectrum Efficiency can be improved up to some extent by having high data rates [1]. The Third Generation Partnership Project Long Term Evolution (3GPP-LTE) has been standardized for the emerging 4th generation (4G) wireless communications. SC-FDMA has been chosen for high data-rate LTE uplink communication systems and OFDMA for downlink communications. SC-FDMA is a single carrier transmission scheme in contrast to OFDMA which is a multicarrier transmission. Single carrier frequency domain multiple access (SC-FDMA) is a promising technique having lower peak to average power ratio (PAPR) and is much more tolerant to amplifier nonlinearities. Similar to OFDMA, it has also carrier frequency offsets caused due to Doppler Effect and dissimilar local oscillators of transmitter and receiver [6]. In such case, the orthogonality among subcarriers are destroyed and inter carrier interference (ICI) and multi access interference (MAI) are generated which will degrade system performance [7].

In this paper, we will deal with the system model of SC-FDMA and will study the effect of CFO on it. A new technique is suggested to avoid the problems and this technique is the hybridization of MMSE Estimation and Kalman Filter [14]. We combined the lower complexity frequency domain scheme with the iterative filter having block of equations in which estimate of process's state is done in a recursive way. Firstly MMSE equalizers are applied as the initial estimation and then Kalman Filter is applied to predict and update the estimated state vector of the system. Better performance is recorded by the hybrid technique.

The rest of this paper is organized as follows- We introduce the system model for the SC-FDMA uplink transmission in Section II. In Section III, the combined technique of MMSE and Kalman Filter is proposed. Simulation results are given in Section IV and concluding remarks are drawn in Section V.

## 2. RELATED WORK

Consider a SC-FDMA system with  $U$  uplink UEs communicating with the base station. A total of  $M$  subcarriers are assumed and each user is assigned a set of  $N$  subcarriers for uplink transmission. Let an  $N \times 1$  column vector denotes the  $N$  QAM symbols of the  $u^{\text{th}}$  UE. Here, one subcarrier is allocated to only one UE. The resulting  $N$  modulated symbols are assembled into blocks and an  $N$ -point DFT is performed to represent input sequence into frequency domain. Here  $d(n)=[d(0), d(1) \dots d(N-1)]$  represents the transmitted data of  $u^{\text{th}}$  user on  $i^{\text{th}}$  subcarrier [2] [3]. Hence,  $N$  point DFT can be expressed as:

$$D(i) = \sum_{n=0}^{N-1} d(n) e^{-\frac{j2\pi}{N}ni} \quad (1)$$

Where  $N$  is the input block size and  $\{d(n):n=0,1 \dots N-1\}$  represents the modulated data symbols.

In subcarrier mapping, each element of  $D(i)$  is mapped to one of  $M$  subcarriers by LFDMA or IFDMA. In localized FDMA, each UE occupies adjacent subcarriers to transmit its symbols whereas in Interleaved FDMA, occupied subcarriers are equidistant from each other. An  $M \times N$  subcarrier mapping matrix  $\Gamma(i)$  for  $u^{\text{th}}$  UE is defined and the frequency domain signal vector after mapping is defined by

$$X(i) = \Gamma(i)D(i) \quad (2)$$

Where

$$\Gamma(i) = \begin{cases} 1, & \text{if } n^{\text{th}} \text{ element is mapped to subcarrier } m \\ 0, & \text{otherwise} \end{cases}$$

After subcarrier mapping, the  $M$ -point IDFT is done to produce time domain signal which can be represented as:

$$x(m) = \frac{1}{M} \sum_{l=0}^{M-1} X(l) e^{\frac{j2\pi}{M}ml} \quad (3)$$

Where  $\{X(l): l=0,1 \dots M-1\}$  represents the frequency domain samples after subcarrier mapping. Finally to avoid inter block interference (IBI), a sufficient long cyclic prefix (CP) is appended at the beginning of each block.

Now we consider that each user experiences a frequency selective channel with impulse response  $h(m)$ , then the received signal at the base station can be expressed as:

$$r(m) = \sum_{u=1}^U [x(m) \otimes h(m) + v(m)] \quad (4)$$

where  $v(m)$  is the AWGN noise on the  $u^{\text{th}}$  user.

But now if we introduce multiple CFOs in the transmitted signal and CFO of the  $u^{\text{th}}$  user is expressed as  $\Delta f$  [15]. Then, after removing cyclic prefix, the received signal can be written as:

$$r(m) = \sum_{u=0}^{U-1} \{ [x(m) \otimes h(m)] \cdot e^{j2\pi\Delta f m} + v(m) \} \quad (5)$$

Then  $M$ -point DFT is done on  $i^{\text{th}}$  subcarrier which can be written as:

$$Y(i) = \frac{1}{M} \sum_{m=0}^{M-1} r(m) \cdot e^{-\frac{j2\pi}{M}mi}$$

$$Y(i) = X(i)H(i)I_0 + \sum_{l=0}^{M-1} X(l)H(l)I_{l-i} + \sum_{u=1}^{U-1} \sum_{l=0, l \neq i}^{M-1} X(l)H(l)I_{l-i} + V(i) \quad (6)$$

Where  $H(l)$  is the frequency domain channel response,  $V(i)$  is the frequency domain AWGN Noise and  $I(l)$  is the interference matrix.

From the above equation, we can see that there is a mixture of original data with the presence of ICI and MAI. Hence, there is a need to remove these components.

## 3. PORPOSED MODELLING

Due to the presence of ICI and MAI in the original signal, the performance of the system degrades. A new technique is to be suggested to avoid the problems caused earlier and this technique is MMSE [12]. It is a lower complexity frequency domain compensation scheme that can achieve equalization and CFO's compensation simultaneously. But still there is a presence of MAI in the system which is to be eliminated. For that purpose we use lower complexity frequency domain scheme with the iterative filter having block of equations in which estimate of process's state is done in a recursive way. Thus, we will define a hybrid technique of MMSE estimator and Kalman filter. We firstly apply MMSE equalizer as the initial estimation to suppress MAI. Then, Kalman filter is applied to predict and update the estimated state vector of the system [13][15].

The received frequency domain signal is given by:

$$Y = I.H.X + V \quad (7)$$

Firstly we will discuss MMSE estimation technique which is given as:

$$W_{MMSE} = \left( HI(HI)^H + \frac{1}{SNR} I_N \right)^{-1} (HI)^H \quad (8)$$

After the subcarrier demapping, the frequency domain estimation of MMSE equalization technique of  $u^{\text{th}}$  user is given by:

$$\hat{Y}^u = W_{MMSE} Y(i) \quad (9)$$

Then, IDFT is done on the output of demapping and later on decoding is applied for detection. After the application of MMSE estimator, still there is a presence of noise which is to be eliminated by Kalman Filter.

The Kalman filter is a set of mathematical equations that provides an efficient recursive means to estimate the state of a process, in a way that minimizes the mean of the squared error. After each iteration of a Kalman filter, there is an updation of the estimate of the state vector of a system and the covariance of that vector based upon the information in a new observation [11]. Thus, the Kalman Filter consists of two steps:

- The Prediction
- The Correction

A state space model of the discrete Kalman Filter can be defined as

$$z(n) = a(n)d(n) + v(n) \quad (10)$$

Here the observation  $z(n)$  has a linear relationship with the desired value  $d(n)$ . We can recursively estimate  $d(n)$  based on the observation of  $z(n)$  and the updated estimation in each recursion is most favorable in the minimum mean square sense. At every update, the error diminishes and the estimate becomes closer to the ideal one. The computation requires offset estimation scheme firstly and then the offset correction is done using Kalman Filter model. The whole process can be summarized as follow-

- Initialize the estimate  $\hat{\varepsilon}(0)$  of the error matrix and the state error to zero.
- Compute the derivative of  $y(n)$  with respect to  $\hat{\varepsilon}(n)$  at  $\hat{\varepsilon}(n-1)$  which is considered as  $H(n)$ .

$$H(n) = \frac{\partial y(x)}{\partial x} = \frac{j2\pi n'}{N} e^{\frac{j2\pi n' \hat{\varepsilon}(n-1)}{N}} x(n) \quad (11)$$

- Calculate the Kalman gain using error variance  $p(n-1)$ ,  $H(n)$  and  $\sigma^2$ .

$$k(n) = p(n-1)H(n)[p(n-1) + \sigma^2]^{-1} \quad (12)$$

- Compute the estimate of  $y^{\wedge}(0)$  using  $x(n)$  and  $\hat{\varepsilon}(n-1)$  and then find the error between the true observation  $y(n)$  and the estimated one  $y^{\wedge}(n)$ .
- Now update the  $\hat{\varepsilon}(n)$  by adding  $k(n)$  to previous estimation  $\hat{\varepsilon}(n-1)$  and then also compute the state error  $p(n)$ .

$$p(n) = [1 - k(n)H(n)]p(n-1)$$

- In this whole procedure, if  $n < N$  then restart the process.

Hence estimate of the frequency offset can be obtained through this recursive iteration procedure. Errors are estimated at every step and can be diminished recursively [17].

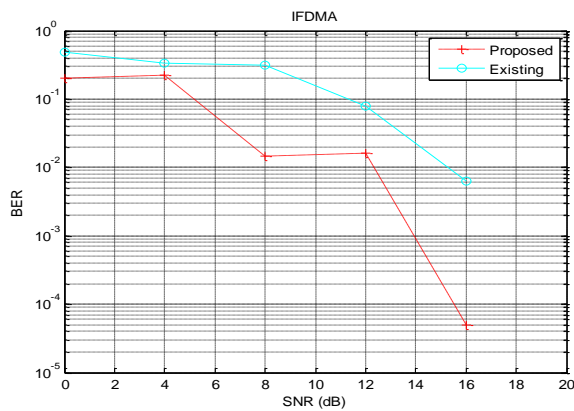
#### 4. RESULTS AND DISCUSSIONS

In this simulation, total of 128 subcarriers is being divided into 4 users having 32 block size for each user [10]. Interleaved subcarrier mapping technique (IFDMA) is being used for this purpose. AWGN channel is made to insert carrier frequency offset (CFO) in the system. Then the proposed technique of MMSE and Kalman Filter is implemented. It is being verified by the Bit Error Rate (BER) graph against different CFOs values with the existing technique of MMSE Estimator. The following table gives a brief description about the simulation parameters:

Parameters	Values
Number of Carriers	128
Modulation	QAM
FFT Size	128
Subcarrier Mapping	IFDMA
Estimation	MMSE
Channel Type	AWGN
Coding	Convolution Coding
Range of SNR	0-16 dB

Table 1: Simulation Parameters

Using these parameters, we can implement the hybrid technique of MMSE Estimator and Kalman Filter in order to enhance the system performance. The various graphs for IFDMA are shown as below:



Similarly we can also plot BER against SNR at different values of CFO. The graphs of various CFO's are shown below:

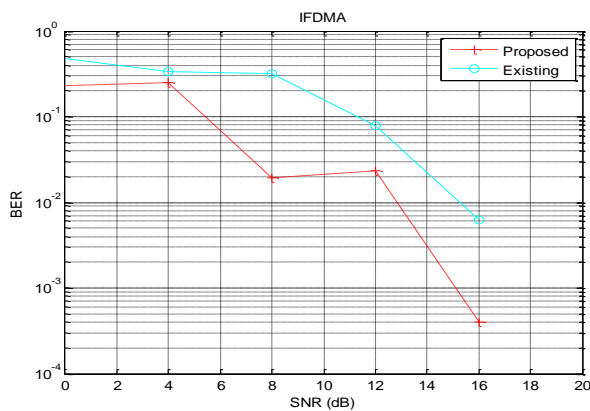


Fig: BER Vs SNR of IFDMA at CFO=0.2

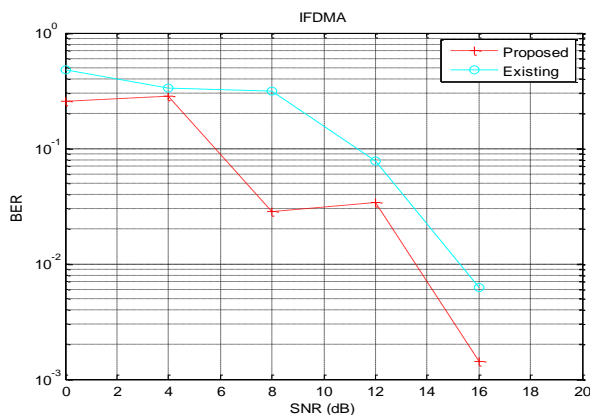


Fig: BER Vs SNR of IFDMA at CFO=0.3

Hence, the following BER plots indicate the improved performance in the proposed technique of the MMSE and Kalman Filter using IFDMA subcarrier mapping. This has been summarized in the following tables.

TABLE II. Comparison Table

SNR	BER		
	CFO=0.1	CFO=0.2	CFO=0.3
4	$2.2 \times 10^{-1}$	$2.5 \times 10^{-1}$	$2.8 \times 10^{-1}$
8	$1.4 \times 10^{-2}$	$1.9 \times 10^{-2}$	$2.8 \times 10^{-2}$
12	$1.6 \times 10^{-2}$	$2.3 \times 10^{-2}$	$3.3 \times 10^{-2}$
16	$5.01 \times 10^{-5}$	$4.01 \times 10^{-4}$	$1.4 \times 10^{-3}$

Table 2: Comparison Table of MMSE and Kalman Filter for IDMA

From the tables, we can interpret that the hybrid technique of MMSE and Kalman Filter gives a better BER ratio and enhances the performance in contrast to the conventional technique of only MMSE estimation. It can also be concluded that the effectiveness of the scheme decreases as the value of CFO increases.

## 5. CONCLUSION

In this paper the system model of SC-FDMA has been discussed. The analysis is focused on the Carrier Frequency Offset introduced in the system. The 3GPP LTE Uplink Simulator is implemented and effect of CFO on uplink transmission is considered. We have seen how the BER varies with SNR values for different CFO values. The comparison is done with existing technique of MMSE Estimator in literature which can attain CFOs compensation and equalization and all together. The hybridization of the techniques can give better results.

An investigation is performed on the proposed algorithm of MMSE Estimator and Kalman Filter. The proposed algorithm proved very efficient in suppressing the carrier frequency offset in the system. The BER value is diminished to some extent causing improvement in the system. As the value of SNR increases, the value of BER decreases. The bit error rate is higher for higher values of frequency offsets.

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